

Oscillations and leptogenesis: what can we learn about right-handed neutrinos? ¹

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Abstract

Inverting the type-I seesaw formula, we reconstruct the mass matrix of the heavy right-handed neutrinos N_i . We analyze how the data on neutrino oscillations affect the structure of this matrix. Under the assumption of hierarchical Dirac-type neutrino masses m_{Di} , we compute the mixing angles among N_i , their masses M_i and the lepton asymmetries ϵ_i generated in their decays. Unless special cancellations take place, one finds $M_i \propto m_{Di}^2$ and the generated baryon-to-photon ratio η_B is much smaller than the observed value. We show that successful baryogenesis via leptogenesis occurs in a unique special case, which corresponds to mass degeneracy and maximal mixing of N_1 and N_2 .

1 Introduction

The smallness of neutrino masses is naturally understood if neutrinos are Majorana particles, since the Majorana mass term can originate only from a five-dimensional operator [1, 2] suppressed by a large energy scale: $LL\phi\phi/\Lambda$, where L and ϕ are the Standard Model (SM) lepton and Higgs doublets, respectively. A very simple and appealing way to generate this operator is the type-I seesaw mechanism [3, 4, 5, 6]. In this case Λ is the mass scale of right-handed (RH) neutrinos, which are SM singlet Majorana particles. The low-energy neutrino mass matrix m is given in terms of the Majorana mass matrix of the RH neutrinos, M_R , and the Dirac mass matrix, m_D , as

$$m = -m_D M_R^{-1} m_D^T . \quad (1)$$

While the elements of m_D are expected to be at or below the electroweak scale (≈ 100 GeV), the characteristic mass scale of RH neutrinos is naturally the GUT or parity breaking scale. For example, in the case of one generation, to obtain a light neutrino mass $m \approx 0.1$ eV one should take $M_R \approx 10^{14}$ GeV.

Understanding the structure of the RH neutrino sector is an important theoretical issue. However, the possibility to investigate such a structure could seem a too arduous experimental task because of the large mass of RH neutrinos. Nevertheless, there are at least two footprints of the seesaw mechanism at accessible energy scales: the light neutrino mass matrix m and the Baryon Asymmetry of the Universe (BAU). In fact, the seesaw

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has a simple and elegant built-in mechanism of production of the BAU: baryogenesis via leptogenesis [7].

The precision in the determination of both the low energy neutrino parameters [8, 9, 10, 11, 12, 13] and the baryon-to photon ratio η_B [14] increased fast in the last few years. The requirement to reproduce these two experimental evidences at the same time is a severe test for the type-I seesaw mechanism. The goal of the present paper is to describe the implications of these constraints for the masses and mixings of RH neutrinos [15].

2 Inverting the seesaw formula

We will consider the light neutrino mass matrix m in the left-handed basis formed by ν_e , ν_μ and ν_τ . In this *flavor basis* the charged lepton mass matrix m_l is diagonal. The Eq.(1) can be rewritten as

$$m = -U_L^\dagger m_D^{diag} U_R (M_R^{diag})^{-1} U_R^T m_D^{diag} U_L^* , \quad (2)$$

where $m_D^{diag} \equiv (m_{D1}, m_{D2}, m_{D3})$ and $M_R^{diag} \equiv (M_1, M_2, M_3)$ are real and positive diagonal matrices with $m_{D1} \leq m_{D2} \leq m_{D3}$ and $M_1 \leq M_2 \leq M_3$. The unitary matrix U_L describes the mismatch between the left-handed rotations diagonalizing m_l and m_D . The unitary matrix U_R describes the mismatch between the RH rotations diagonalizing m_D and M_R .

In the RH basis in which the matrix U_R is absorbed in M_R , inverting the seesaw formula (2) one obtains

$$M_R^{-1} \equiv U_R (M_R^{diag})^{-1} U_R^T = - \begin{pmatrix} \frac{\hat{m}_{ee}}{m_{D1}^2} & \frac{\hat{m}_{e\mu}}{m_{D1}m_{D2}} & \frac{\hat{m}_{e\tau}}{m_{D1}m_{D3}} \\ \cdots & \frac{\hat{m}_{\mu\mu}}{m_{D2}^2} & \frac{\hat{m}_{\mu\tau}}{m_{D2}m_{D3}} \\ \cdots & \cdots & \frac{\hat{m}_{\tau\tau}}{m_{D3}^2} \end{pmatrix} , \quad (3)$$

where

$$\hat{m} \equiv U_L m U_L^T . \quad (4)$$

For simplicity, we denote the entries of \hat{m} with e, μ, τ indexes, even though \hat{m} is the light neutrino mass matrix in a basis rotated with respect to the flavor basis.

Even if the matrix m were completely known from experiments (see discussion in section 3), one cannot infer the masses of RH neutrinos unless some assumption is made on the Dirac mass matrix m_D . In this paper we will analyze only the case of hierarchical mass spectrum for the neutrino Dirac masses:

$$m_{D1} \ll m_{D2} \ll m_{D3} . \quad (5)$$

This choice is motivated by the assumption that m_D is analogue to the Dirac mass matrices of quarks or charged leptons. This is the case in many theories with quark-lepton symmetry (for example, in minimal $SO(10)$ one has $m_D = m_u$ at GUT scale). In other words, we assume that the hierarchy among different generation of Yukawa

couplings is a property which holds also in the neutrino sector. Other studies of the seesaw mechanism and leptogenesis with hierarchical neutrino Dirac masses can be found in [16, 17, 18, 19, 20, 21, 22].

To quantify the strength of the hierarchy that one should expect among m_{Di} , we report in Table 1 the approximate values of charged lepton and quark masses at the renormalization scale 10^9 GeV (SM case [23]). In fact, we are interested in the value of neutrino Dirac masses at the scale of RH neutrino masses. For numerical estimates, we will use for m_{Di} the values of up-quark masses (the case of minimal $SO(10)$) given in Table 1.

	1st generation	2nd generation	3rd generation
up-type quarks	1 MeV	400 MeV	100 GeV
down-type quarks	3 MeV	50 MeV	2 GeV
charged leptons	0.5 MeV	100 MeV	2 GeV
neutrinos	m_{D1}	m_{D2}	m_{D3}

Table 1: Approximate values of SM quark and charged lepton masses renormalized at 10^9 GeV [23].

3 Basic features of low energy neutrino mass matrix

The r.h.s. of Eq.(3) depends on m , U_L and m_{Di} . At present, we have direct experimental access only to the Majorana mass matrix of light neutrinos, m . It can be written in terms of the observables as

$$m = U_{PMNS}^* m^{diag} U_{PMNS}^\dagger, \quad (6)$$

where $m^{diag} \equiv diag(m_1, m_2, m_3)$ and

$$U_{PMNS} = U(\theta_{12}, \theta_{23}, \theta_{13}, \delta) \cdot K_0, \quad K_0 = diag(e^{i\rho}, 1, e^{i\sigma}). \quad (7)$$

Here δ is the CP-violating Dirac phase and ρ and σ are the two CP-violating Majorana phases. The matrix m should satisfy a number of experimental constraints. From the solar, atmospheric, accelerator and reactor neutrino experiments we take the following input (at 90% C.L.) [8, 9, 10, 11, 12, 13]:

$$\begin{aligned} \Delta m_{sol}^2 \equiv \Delta m_{12}^2 &= (7.1 \begin{smallmatrix} +1.9 \\ -1.1 \end{smallmatrix}) \cdot 10^{-5} \text{ eV}^2, & \tan^2 \theta_{12} &= 0.40^{+0.12}_{-0.09}; \\ \Delta m_{atm}^2 \equiv \Delta m_{23}^2 &= (2.0 \begin{smallmatrix} +1.1 \\ -0.7 \end{smallmatrix}) \cdot 10^{-3} \text{ eV}^2, & \tan \theta_{23} &= 1 \begin{smallmatrix} +0.35 \\ -0.25 \end{smallmatrix}; \\ \sin \theta_{13} &\lesssim 0.2. \end{aligned} \quad (8)$$

A significant freedom in the structure of the mass matrix m still exists due to the unknown absolute mass scale m_1 and CP-violating phases ρ and σ [24, 25]. In spite of this freedom, a generic feature of the matrix m emerges: all its elements are of the same order (within a factor of 10 or so of each other), except in some special cases. The reason for this is twofold:

(a) a relatively weak hierarchy between the mass eigenvalues:

$$\frac{m_2}{m_3} \geq R \equiv \sqrt{\frac{\Delta m_{sol}^2}{\Delta m_{atm}^2}} = 0.19_{-0.05}^{+0.07} ; \quad (9)$$

(b) two large mixing angles θ_{12} and θ_{23} .

A strong hierarchy among certain elements of m can be realized for specific values of m_1 , ρ and σ . For example, in the case of normal mass hierarchy ($m_1 \ll m_2, m_3$), the e -row elements of m (that is m_{ee} , $m_{e\mu}$ and $m_{e\tau}$) are smaller than $\mu\tau$ -block ones by a factor of order R or s_{13} . An e -row element can vanish for specific values of σ .

Let us consider how these features of m reflect on \hat{m} . The neutrino masses m_i are of course basis-independent. On the contrary, the mixing angles and the phases in \hat{m} take values $\hat{\theta}_{ij}$, $\hat{\delta}$, $\hat{\rho}$, $\hat{\sigma}$ different with respect to flavor basis. As a consequence, the condition (a) applies both for m and \hat{m} , while (b) can be no longer valid in the rotated matrix \hat{m} , if large mixing angles are contained in U_L (see Eq.(4)). However, since the matrix U_L is the leptonic analogue of the quark CKM mixing matrix, by analogy one can assume that U_L is close to the unit matrix (does not contain large mixing angles). Therefore one expects that, as in the case of m , all the elements of \hat{m} are of the same order, apart from special cases.

Even if there are large mixings in U_L , the vanishing of some elements in \hat{m} requires special cancellations. For example, in the case of normal hierarchy, the e -row elements (\hat{m}_{ee} , $\hat{m}_{e\mu}$, $\hat{m}_{e\tau}$) vanish if both $\hat{\theta}_{12}$ and $\hat{\theta}_{13}$ are vanishing. This requires that U_L in Eq.(4) contains a large $1 - 2$ mixing angle which cancels exactly the large solar mixing in m . In the following we assume that these cancellations do not take place. We will further comment on the case of large left-handed Dirac-type mixing in section 7.

Using the low energy data we can study, in particular, the condition $\hat{m}_{ee} \rightarrow 0$, which will turn out to be crucial in the following discussion. If $U_L = \mathbb{1}$, $\hat{m}_{ee} = m_{ee}$. Using Eq.(6) and the standard parameterization for the matrix U one obtains

$$m_{ee} = \cos^2 \theta_{13} (m_1 e^{-2i\rho} \cos^2 \theta_{12} + m_2 \sin^2 \theta_{12}) + \sin^2 \theta_{13} e^{2i(\delta-\sigma)} m_3 .$$

The condition $m_{ee} \rightarrow 0$ is satisfied for

$$\tan^2 \theta_{13} \approx - \frac{m_1 e^{-2i\rho} \cos^2 \theta_{12} + m_2 \sin^2 \theta_{12}}{e^{2i(\delta-\sigma)} m_3} . \quad (10)$$

In the limit $\sin \theta_{13} = 0$, Eq.(10) implies $\rho \approx \pi/2$ and

$$m_1 \approx \frac{\tan^2 \theta_{12} \sqrt{\Delta m_{sol}^2}}{\sqrt{1 - \tan^4 \theta_{12}}} \approx (3 - 4) \cdot 10^{-3} \text{eV} . \quad (11)$$

This corresponds to normal mass hierarchy. Non-zero $\sin \theta_{13}$ shifts the value of m_1 corresponding to $m_{ee} \rightarrow 0$. Taking into account the present upper bound on $\sin \theta_{13}$ (Eq.(8)), we find that the relation (10) can be satisfied for $m_1 \lesssim 0.02$ eV. Notice that larger values of m_1 are forbidden because θ_{12} is far from the maximal value $\theta_{12}^{max} \equiv \pi/4$.

If $U_L \neq \mathbb{1}$, the condition $\hat{m}_{ee} \rightarrow 0$ is satisfied if the angles $\hat{\theta}_{ij}$ and the phases $\hat{\delta}$, $\hat{\rho}$ and $\hat{\sigma}$ fulfill Eq.(10). This is possible also for mass spectra different from normal hierarchy. In

this case $m_2 \approx m_1 \gg \sqrt{\Delta m_{sol}^2}$ and Eq.(10) can be satisfied, e.g., for $\hat{\theta}_{13} = 0$, $\hat{\theta}_{12} = \pi/4$ and $\hat{\rho} = \pi/2$. These values of mixing angles can be obtained for $U_L \approx U_{CKM}$ [26].

Notice that the neutrinoless 2β decay experiments [27, 28] restrict the ee -element of the matrix m :

$$|m_{ee}| < (0.35 \div 1.3) \text{ eV} \quad (90\% \text{ C.L.}) .$$

If future experiments will find a positive signal, this will imply that m_{ee} is not very small (unless non-standard mechanism contribute to neutrinoless 2β -decay rate [29]). In this case the condition $\hat{m}_{ee} \rightarrow 0$ could be satisfied only for non-negligible rotations in U_L .

4 Mass spectrum and mixing of RH neutrinos

Let us compute the eigenvalues and the mixing angles of the matrix M_R^{-1} defined in Eq.(3).

Generic case.

The denominators in the r.h.s. of Eq.(3) are strongly hierarchical. As a consequence, unless a special suppression of \hat{m}_{ee} takes place, the largest eigenvalue of M_R^{-1} is given, to a very good approximation, by the dominant 11-element:

$$M_1 \approx \frac{1}{|(M_R^{-1})_{11}|} = \frac{m_{D1}^2}{|\hat{m}_{ee}|} . \quad (12)$$

The second largest eigenvalue of M_R^{-1} can be obtained from the dominant (12)-block of the matrix (3), just by dividing its determinant by $(M_R^{-1})_{11}$. The mass M_2 is then the inverse of this eigenvalue:

$$M_2 \approx \frac{|(M_R^{-1})_{11}|}{|(M_R^{-1})_{11}(M_R^{-1})_{22} - (M_R^{-1})_{12}^2|} = \frac{m_{D2}^2 |\hat{m}_{ee}|}{|d_{12}|} , \quad (13)$$

where

$$d_{12} \equiv \hat{m}_{ee} \hat{m}_{\mu\mu} - \hat{m}_{e\mu}^2 .$$

The Eq.(13) is reliable as far as the subdeterminant d_{12} is not vanishing. The smallest eigenvalue of M_R^{-1} can be found from the condition

$$(m_{D1} m_{D2} m_{D3})^2 = m_1 m_2 m_3 M_1 M_2 M_3$$

which is obtained by taking the determinants of both sides of Eq.(1). This yields

$$M_3 \approx \frac{m_{D3}^2 |d_{12}|}{m_1 m_2 m_3} . \quad (14)$$

Thus, in the generic case the RH neutrinos have a very strong mass hierarchy: $M_1 \propto m_{D1}^2$, $M_2 \propto m_{D2}^2$, $M_3 \propto m_{D3}^2$. Assuming $U_L \approx \mathbb{1}$, the numerical values of M_i are functions of low energy data and m_{Di} only. One finds [15]

$$\begin{aligned} M_1 &\approx (1 - 500) \text{ TeV} \left(\frac{m_{D1}}{1 \text{ MeV}} \right)^2 , \\ M_2 &\approx (0.2 - 6) \cdot 10^9 \text{ GeV} \left(\frac{m_{D2}}{400 \text{ MeV}} \right)^2 , \\ M_3 &\gtrsim 5 \cdot 10^{12} \text{ GeV} \left(\frac{m_{D3}}{100 \text{ GeV}} \right)^2 , \end{aligned}$$

where the given ranges reflect our ignorance on the type of mass spectrum of light neutrinos.

The matrix M_R is diagonalized, to a high accuracy, by

$$U_R \approx \begin{pmatrix} 1 & -\left(\frac{\hat{m}_{e\mu}}{\hat{m}_{ee}}\right)^* \frac{m_{D1}}{m_{D2}} & \left(\frac{d_{23}}{d_{12}}\right)^* \frac{m_{D1}}{m_{D3}} \\ \left(\frac{\hat{m}_{e\mu}}{\hat{m}_{ee}}\right) \frac{m_{D1}}{m_{D2}} & 1 & -\left(\frac{d_{13}}{d_{12}}\right)^* \frac{m_{D2}}{m_{D3}} \\ \left(\frac{\hat{m}_{e\tau}}{\hat{m}_{ee}}\right) \frac{m_{D1}}{m_{D3}} & \left(\frac{d_{13}}{d_{12}}\right) \frac{m_{D2}}{m_{D3}} & 1 \end{pmatrix} \cdot K, \quad (15)$$

where

$$d_{23} \equiv \hat{m}_{e\mu}\hat{m}_{\mu\tau} - \hat{m}_{\mu\mu}\hat{m}_{e\tau}, \quad d_{13} \equiv \hat{m}_{ee}\hat{m}_{\mu\tau} - \hat{m}_{e\mu}\hat{m}_{e\tau}$$

and

$$K = \text{diag}(e^{-i\phi_1/2}, e^{-i\phi_2/2}, e^{-i\phi_3/2}). \quad (16)$$

The differences between the Majorana phases ϕ_i of RH neutrinos have physical meaning, analogously to the case of light neutrinos (see Eq.(7)). As can be seen from Eq.(15), all the three RH mixing angles are very small in the generic case ($\lesssim m_{D1}/m_{D2}$, m_{D2}/m_{D3}). If also the left-handed mixing angles in U_L are small, still one can obtain a strong mixing in the low-energy sector. This is the so-called “seesaw enhancement” of the leptonic mixing [30]. The reason for this enhancement can be readily understood. Indeed, small mixing in m_D and M_R is related to the hierarchical structures of these matrices; however, in the seesaw formula (1) these hierarchies act in the opposite directions and largely compensate each other, leading to a “quasi-democratic” m and thus to large mixing in the low-energy sector.

Special case $\hat{m}_{ee} \rightarrow 0$.

When

$$|\hat{m}_{ee}| \ll \frac{m_{D1}}{m_{D2}} |\hat{m}_{e\mu}|, \quad (17)$$

the 12-block of M_R^{-1} in Eq.(3) is dominated by the off-diagonal entries and, to a good approximation, the two lightest RH neutrinos have opposite CP-parity and equal masses:

$$M_1 \approx M_2 \approx \frac{1}{|(M_R^{-1})_{12}|} \approx \frac{m_{D1}m_{D2}}{|\hat{m}_{e\mu}|}, \quad M_3 \approx \frac{m_{D3}^2 |\hat{m}_{e\mu}|^2}{m_1 m_2 m_3}. \quad (18)$$

Notice that M_1 is increased by a factor $\sim m_{D2}/m_{D1}$ with respect to the generic case (Eq.(12)). Assuming $U_L \approx \mathbb{1}$, one obtains [15]

$$\begin{aligned} M_{1,2} &\approx 9 \cdot 10^7 \text{ GeV} \left(\frac{m_{D1}}{1 \text{ MeV}} \right) \left(\frac{m_{D2}}{400 \text{ MeV}} \right), \\ M_3 &\approx 10^{14} \text{ GeV} \left(\frac{m_{D3}}{100 \text{ GeV}} \right)^2. \end{aligned}$$

These predictions are more precise than in the generic case, since the light neutrino mass spectrum is fixed by the condition $m_{ee} \rightarrow 0$ (see Eq.(11)).

The RH 1 – 2 mixing is nearly maximal while the other mixing angles remain very small:

$$U_R \approx \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\left(\frac{d_{23}}{\hat{m}_{e\mu}^2}\right)^* \frac{m_{D1}}{m_{D3}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\left(\frac{\hat{m}_{e\tau}}{\hat{m}_{e\mu}}\right)^* \frac{m_{D2}}{m_{D3}} \\ -\frac{\hat{m}_{e\tau}}{\sqrt{2}\hat{m}_{e\mu}} \frac{m_{D2}}{m_{D3}} & \frac{\hat{m}_{e\tau}}{\sqrt{2}\hat{m}_{e\mu}} \frac{m_{D2}}{m_{D3}} & 1 \end{pmatrix} \cdot K . \quad (19)$$

The matrix of phases K is given in Eq. (16) and one has, in particular, $\phi_1 - \phi_2 \approx \pi$. Thus, the RH neutrinos N_1 and N_2 are quasi-degenerate, have nearly opposite CP-parities and almost maximal mixing (1 – 2 level crossing). The third RH neutrino N_3 is much heavier and weakly mixed with the first two.

Special case $d_{12} \rightarrow 0$.

Let us consider the case in which the (11)-element of the matrix M_R^{-1} in Eq.(3) is still the dominant one (as in the generic case), but the (12)-subdeterminant of M_R^{-1} is very small. Then $(M_R)_{33}$, which is proportional to this subdeterminant, is suppressed. The condition $(M_R)_{33} \ll (M_R)_{23}$ can be written as

$$|d_{12}| \ll \frac{m_{D2}}{m_{D3}} |d_{13}| . \quad (20)$$

In this case M_1 is still given by Eq.(12), but the (23)-block of M_R is dominated by its off-diagonal entry. This yields

$$M_2 \approx M_3 \approx |(M_R)_{23}| = \frac{m_{D2}m_{D3}}{m_1m_2m_3} |d_{13}| . \quad (21)$$

The matrix U_R is similar to the one in Eq.(19) but with maximal mixing in the 2 – 3 sector [15].

Special case $\hat{m}_{ee} \rightarrow 0$ & $d_{12} \rightarrow 0$.

Consider the case when

$$|\hat{m}_{ee}| \ll \frac{m_{D1}}{m_{D3}} |\hat{m}_{e\tau}| , \quad |\hat{m}_{e\mu}| \ll \frac{m_{D2}}{m_{D3}} |\hat{m}_{e\tau}|, \quad \frac{m_{D1}}{m_{D2}} |\hat{m}_{\mu\mu}| . \quad (22)$$

Then both \hat{m}_{ee} and d_{12} are vanishing. The (13)- and (22)-elements of M_R^{-1} are the dominant ones (see Eq.(3)). Two RH neutrinos form a quasi-degenerate pair with almost maximal mixing and opposite CP-parities and the third neutrino has small mixing with the other two (of order m_{D1}/m_{D2} or m_{D2}/m_{D3}). The masses of these doublet and singlet states are given by

$$M_d \approx \frac{1}{|(M_R^{-1})_{13}|} \approx \frac{m_{D1}m_{D3}}{|\hat{m}_{e\tau}|} , \quad M_s \approx \frac{1}{|(M_R^{-1})_{22}|} \approx \frac{m_{D2}^2}{|\hat{m}_{\mu\mu}|} . \quad (23)$$

Since $m_{D1}m_{D3} \sim m_{D2}^2$, all the three masses are of the same order ($\sim 10^{10}$ GeV). The explicit form of U_R for this case can be found in [15].

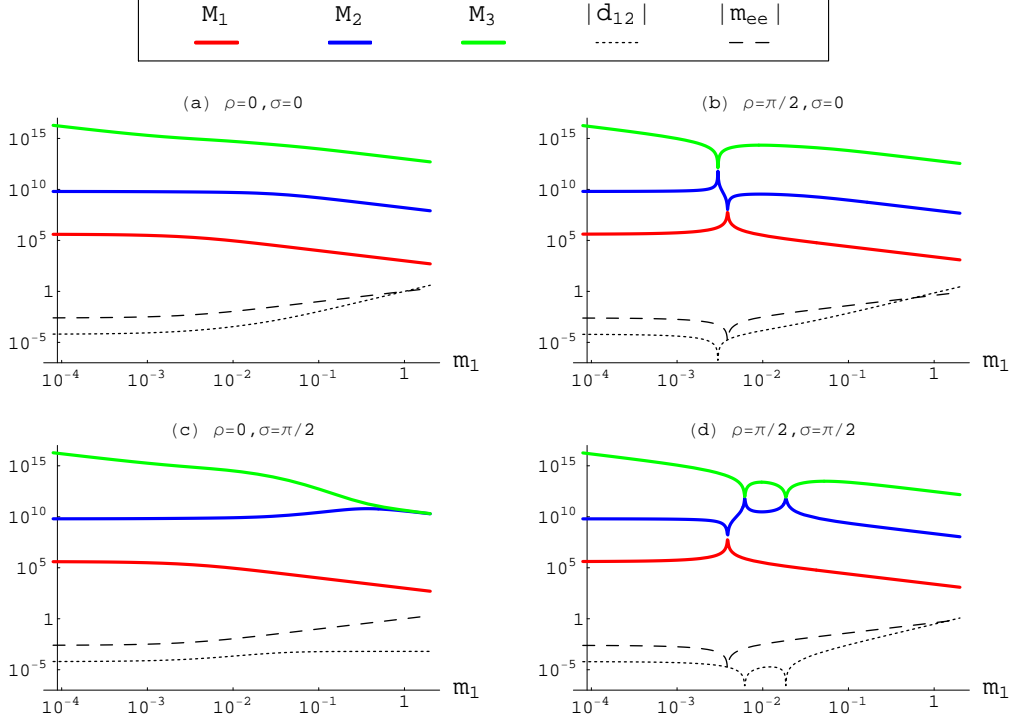


Figure 1: The masses of RH neutrinos M_i in GeV as functions of the light neutrino mass m_1 in eV (solid thick lines), for different values of the Majorana phases of light neutrinos, ρ and σ . We assume normal mass ordering; $U_L = \mathbb{1}$; $s_{13} = 0$; best fit values of solar and atmospheric mixing angles and mass squared differences (Eq.(8)); values of m_{Di} given by the up-type quark masses in Table 1. Also shown are $|d_{12}| \equiv |m_{ee}m_{\mu\mu} - m_{e\mu}^2|$ in eV^2 (dotted thin line) and $|m_{ee}|$ in eV (dashed thin line) as functions of m_1 .

Notice that in all the three special cases, the mass-degeneracy of two RH neutrinos is associated with almost maximal mixing between them and opposite relative CP-parity. In fact, when mass hierarchies in m_D and M_R do not compensate, a strongly off-diagonal structure of M_R is necessary for the seesaw enhancement of lepton mixing [30].

The features of RH neutrino mass spectrum can be seen in Fig.1, where, assuming $\hat{m} = m$, we show the dependence of the RH neutrino masses on the lightest mass m_1 , for different values of the Majorana phases of the light neutrinos ρ and σ . One sees immediately that the crossing points where $M_1 \approx M_2$ correspond to $m_{ee} \rightarrow 0$. The other possible level crossing ($M_2 \approx M_3$) is realized when $d_{12} \rightarrow 0$.

5 Baryogenesis via leptogenesis

Let us consider the constraints on the seesaw parameters coming from the requirement of successful thermal leptogenesis. We assume that a lepton asymmetry ϵ_i is generated by the CP-violating out-of-equilibrium decays of the RH neutrino N_i in the early Universe [7]:

$$\epsilon_i = \frac{\Gamma(N_i \rightarrow L\phi) - \Gamma(N_i \rightarrow \bar{L}\bar{\phi})}{\Gamma(N_i \rightarrow L\phi) + \Gamma(N_i \rightarrow \bar{L}\bar{\phi})} ,$$

where L and ϕ are the SM lepton and Higgs doublets. The lepton asymmetry is then converted to a baryon asymmetry through the sphaleron processes [31], thus explaining the baryon asymmetry of the Universe. We will use the recent experimental value of the baryon-to-photon ratio [14],

$$\eta_B = (6.5 \pm_{0.3}^{0.4}) \cdot 10^{-10} . \quad (24)$$

The lepton number asymmetry ϵ_i can be written as [32, 33, 34, 35, 36]:

$$\epsilon_i = \frac{1}{8\pi} \sum_{k \neq i} f \left(\frac{M_k^2}{M_i^2} \right) \frac{\text{Im}[(h^\dagger h)_{ik}^2]}{(h^\dagger h)_{ii}} . \quad (25)$$

Here h is the matrix of neutrino Yukawa couplings in the basis where M_R is diagonal with real and positive eigenvalues. Using the relation $h \equiv m_D/v = (U_L^\dagger m_D^{diag} U_R)/v$ (where $v = 174$ GeV is the electroweak VEV) we can write

$$h^\dagger h = \frac{1}{v^2} U_R^\dagger (m_D^{diag})^2 U_R . \quad (26)$$

In the SM the function f in Eq. (25) is given by

$$f(x) = \sqrt{x} \left[\frac{2-x}{1-x} - (1+x) \log \left(\frac{1+x}{x} \right) \right] . \quad (27)$$

This expression is valid for $|M_i - M_j| \gg \Gamma_i + \Gamma_j$, where Γ_i is the decay width of the i th RH neutrino, given at tree level by

$$\Gamma_i = \frac{(h^\dagger h)_{ii}}{8\pi} M_i .$$

In the limit of the quasi-degenerate neutrinos ($x = M_j^2/M_i^2 \rightarrow 1$), one formally obtains from (27)

$$f(x) \approx \frac{1}{1-x} \approx \frac{M_i}{2(M_i - M_j)} \rightarrow \infty . \quad (28)$$

However, in reality the enhancement of the asymmetry is limited by the decay widths Γ_i and is maximized when $|M_i - M_j| \sim \Gamma_i + \Gamma_j$ [37, 38, 39].

The baryon-to-photon ratio can be written as [40]

$$\eta_B \simeq 0.01 \sum_i \epsilon_i \cdot \kappa_i ,$$

where the factors κ_i describe the washout of the produced lepton asymmetry ϵ_i due to various lepton number violating processes. In the domain of the parameter space which is of interest to us, they depend mainly on the effective mass parameters

$$\tilde{m}_i \equiv \frac{v^2 (h^\dagger h)_{ii}}{M_i} = \frac{[U_R^\dagger (m_D^{diag})^2 U_R]_{ii}}{M_i} . \quad (29)$$

For 10^{-2} eV $< \tilde{m}_1 < 10^3$ eV, the washout factor κ_1 can be well approximated by [41]

$$\kappa_1(\tilde{m}_1) \simeq 0.3 \left(\frac{10^{-3} \text{ eV}}{\tilde{m}_1} \right) \left(\log \frac{\tilde{m}_1}{10^{-3} \text{ eV}} \right)^{-0.6} . \quad (30)$$

When $M_1 \ll M_{2,3}$, only the decays of the lightest RH neutrino N_1 are relevant for producing the baryon asymmetry η_B , since the lepton asymmetry generated in the decays of the heavier RH neutrinos is washed out by the L -violating processes involving N_1 's, which are very abundant at high temperatures $T \sim M_{2,3}$. At the same time, at $T \sim M_1$ the heavier neutrinos N_2 and N_3 have already decayed and so cannot wash out the asymmetry produced in the decays of N_1 .

For a recent systematic study of thermal leptogenesis with a detailed analysis of washout effects see [42].

6 A unique structure for successful thermal leptogenesis

Let us compute the value of η_B generated through the decays of RH neutrinos in the different cases discussed in section 4.

Generic case.

From Eqs.(29), (12) and (15) we get

$$\tilde{m}_1 \approx \frac{|\hat{m}_{ee}|^2 + |\hat{m}_{e\mu}|^2 + |\hat{m}_{e\tau}|^2}{|\hat{m}_{ee}|} . \quad (31)$$

Assuming $U_L \approx \mathbb{1}$ and using low energy data, it turns out [15] that $\tilde{m}_1 \gtrsim \sqrt{\Delta m_{sol}^2}$, so that Eq.(30) implies $\kappa_1 \lesssim 0.02$. From Eqs.(25)-(27) and (12)-(15), we obtain the following expression for the lepton asymmetry:

$$\epsilon_1 \approx \frac{3}{16\pi} \frac{m_{D1}^2}{v^2} \cdot I(\hat{m}_{\alpha\beta}) ,$$

where $I(\hat{m}_{\alpha\beta})$ is an order one function of the elements of \hat{m} . Then the produced baryon-to-photon ratio is given, up to a factor of order one, by

$$\eta_B \simeq 0.01 \cdot \epsilon_1 \cdot \kappa_1 \simeq 4 \cdot 10^{-16} \left(\frac{m_{D1}}{1 \text{ MeV}} \right)^2 \left(\frac{\kappa_1}{0.02} \right) .$$

To reproduce the observed value of η_B , one would need $m_{D1} \sim 1$ GeV. Thus, a successful leptogenesis requires $m_{D1} \sim m_{D2}$, which contradicts our assumption of a strong hierarchy between the eigenvalues of m_D and goes contrary to the simple GUT expectations. Therefore, the generic case does not lead to successful leptogenesis.

Special case $\hat{m}_{ee} \rightarrow 0$.

Since N_1 and N_2 are quasi-degenerate and almost maximally mixed, ϵ_1 and ϵ_2 are almost equal. The dominant contribution to $\epsilon_{1,2}$ is given by (see Eqs.(19), (25), (26) and (28))

$$\epsilon_1 \approx \epsilon_2 \approx \frac{1}{16\pi} \frac{M_1}{M_1 - M_2} \frac{\text{Im}[(h^\dagger h)_{12}^2]}{(h^\dagger h)_{11}} \approx \frac{1}{16\pi} \frac{m_{D2}^2}{v^2} \xi, \quad (32)$$

where

$$\xi = \frac{M_1}{M_1 - M_2} \sin(\phi_1 - \phi_2). \quad (33)$$

The enhancement due to the quasi-degeneracy of N_1 and N_2 competes with the suppression due to their almost opposite CP-parities: $(\phi_1 - \phi_2) \approx \pi$. Starting from Eq.(3) and performing a detailed computation of the mass splitting and of the deviation of $\sin(\phi_1 - \phi_2)$ from zero, one finds

$$\xi \approx \frac{4k \tan \Delta}{(1+k)^2 + (1-k)^2 \tan^2 \Delta}, \quad (34)$$

where

$$k \equiv \frac{m_{D1}^2 |\hat{m}_{\mu\mu}|}{m_{D2}^2 |\hat{m}_{ee}|}, \quad \Delta \equiv \frac{1}{2} \arg \frac{\hat{m}_{e\mu}^2}{\hat{m}_{ee} \hat{m}_{\mu\mu}}. \quad (35)$$

For $|1 - k| \ll 1/\tan \Delta$, Eq.(34) gives $\xi \approx \tan \Delta$, so that for $\Delta \simeq \pi/2$ a significant enhancement of the asymmetries $\epsilon_{1,2}$ can be achieved.

Because of almost maximal 1-2 RH mixing, both N_1 and N_2 interact with the thermal bath mainly via the Yukawa coupling m_{D2}/v . This, in contrast with the generic case, implies $\epsilon_1 \propto m_{D2}^2$ instead of m_{D1}^2 , but also washout effects much stronger. In fact, from Eqs.(29), (18) and (19) we obtain

$$\tilde{m}_1 \approx \tilde{m}_2 \approx \frac{m_{D2}}{m_{D1}} \frac{|\hat{m}_{e\mu}|^2 + |\hat{m}_{e\tau}|^2}{2|\hat{m}_{e\mu}|}.$$

Assuming $U_L \approx 1$ and using for m_{Di} the values given in Table 1 for up-type quarks, we find [15] $\tilde{m}_1 \approx 1.5$ eV and thus Eq.(30) implies $\kappa_1 \approx \kappa_2 \approx 6 \cdot 10^{-5}$.

Combining Eqs.(30) and (32) and taking into account the restriction $M_2 - M_1 \gtrsim \Gamma_1$ (see section 5), one finally obtains [15]

$$\eta_B \approx 0.01 \cdot 2\epsilon_1 \kappa_1 \lesssim 2 \cdot 10^{-8} \left(\frac{400m_{D1}}{m_{D2}} \right)^2 \left[1 + 0.14 \log \left(\frac{m_{D2}}{400m_{D1}} \right) \right]^{-0.6}.$$

The value (24) of η_B can be reproduced for $m_{D1}/m_{D2} \gtrsim 2 \cdot 10^{-3}$. This corresponds [15] to a relative splitting $(M_2 - M_1)/M_1 \lesssim 10^{-5}$.

Thus, in spite of strong washout effects, a sufficiently large baryon asymmetry can be generated in this special case, due to the enhancement related to the strong degeneracy of the RH neutrinos. For this to occur, not only the level crossing condition ($\hat{m}_{ee} \rightarrow 0$) has to be satisfied, but also the conditions $\Delta \approx \pi/2$ and $k \approx 1$ should be fulfilled, where Δ and k are defined in Eq.(35). All these requirements are consistent with the low energy neutrino data. We have checked these analytic results by precise numerical calculations.

Other special cases.

In the special case $d_{12} \rightarrow 0$ ($M_1 \ll M_2 \approx M_3$), the produced lepton asymmetry is

dominated by the decays of N_1 . The RH mixing angles are larger than in the generic case, but the contributions to ϵ_1 from diagrams with N_2 or N_3 in the loop cancel each other because of their opposite CP-parity [15]. The final asymmetry is much smaller than the required value.

In the special case $\hat{m}_{ee} \rightarrow 0$ & $d_{12} \rightarrow 0$, since the three RH neutrinos have similar masses, the decays of all three N_i 's can contribute to the produced lepton asymmetry. One finds [15] that some of the ϵ_i 's can be large, but correspondingly washout effects are very strong, because the large 1 – 3 RH mixing implies that the pair of maximally mixed RH neutrinos interacts with the thermal bath via the order one coupling m_{D3}/v . As a consequence, leptogenesis is unsuccessful.

7 Stability of the result

In the previous section we have computed the baryon asymmetry produced through the decays of RH neutrinos, in the framework of type-I seesaw mechanism with hierarchical Dirac masses m_{Di} and small left-handed Dirac-type mixing U_L . We have found that the unique possibility to obtain successful thermal leptogenesis is the special case $\hat{m}_{ee} \rightarrow 0$. Now we want to give some comments and to make checks on the stability of this result.

1) Supersymmetry.

The successful special case works also in the SUSY version, since the mass scale $M_1 \approx M_2 \sim 10^8$ GeV can be easily smaller than the reheating temperature required to avoid gravitino overproduction [43, 44, 45].

In the Minimal Supersymmetric SM the electroweak VEV v in Eq.(26) should be replaced with $v \sin \beta$. However, for $\tan \beta \gtrsim 3$, this corresponds to a very small rescaling of Yukawa couplings. As a consequence, the estimation of the lepton asymmetry is not significantly modified with respect to the SM case.

In some supersymmetric scenarios, U_L and m_{Di} can be probed in lepton flavor violating (LFV) decays like $\mu \rightarrow e\gamma$ or $\tau \rightarrow \mu\gamma$ [46, 47, 48, 49, 50]. If $U_L = \mathbb{1}$, these decays are strongly suppressed and will not be observed. On the contrary, for $U_L \approx U_{CKM}$ one finds the predicted branching ratios to be close to the experimental upper bounds, provided that the slepton masses are of the order of $(100 \div 200)$ GeV and the neutrino Dirac masses m_{Di} take values of the order of quark masses. Therefore, if future experiments find a signal close to the present upper bounds, this will not require large rotations in U_L . In any case the successful special case is not constrained by LFV bounds, since the condition $\hat{m}_{ee} \rightarrow 0$ can be satisfied both for $U_L = \mathbb{1}$ and $U_L \neq \mathbb{1}$.

2) Flavor symmetries in the RH sector.

The existence of a pair of degenerate and maximally mixed RH neutrinos may well be the consequence of some flavor symmetry in the RH sector, like $SU(2)_H$ under which N_1 and N_2 transform like a doublet [51].

It is well known [52, 53] that a pseudo-Dirac structure of the light neutrino mass matrix is useful to explain large mixing, especially in the context of models with inverted mass hierarchy. We have found that also in the RH neutrino sector an approximate pseudo-Dirac structure of the mass matrix can have important consequences, both for mixing [30] and lepton asymmetry enhancement.

3) Radiative corrections.

Let us discuss the renormalization group equation (RGE) evolution of the seesaw mass matrices.

The structure of the effective mass matrix m is stable under the SM (or MSSM) radiative corrections [54, 55, 56, 57]. The corrections to its matrix elements can be written as

$$\Delta m_{\alpha\beta} \sim (\epsilon_\alpha + \epsilon_\beta) m_{\alpha\beta} , \quad (36)$$

where ϵ_α ($\lesssim 10^{-2}$) describes the effect of the Yukawa coupling of the charged lepton l_α . The Eq.(36) implies that both m_{ee} and $(m_{ee}m_{\mu\mu} - m_{e\mu}^2)$ receive small corrections proportional to themselves: if they are very small at the electroweak scale, they remain very small also at the seesaw scale (the mass scale of RH neutrinos). Therefore, if $U_L = \mathbb{1}$, the level crossing conditions $\hat{m}_{ee} \rightarrow 0$ and $d_{12} \rightarrow 0$ can be tested with low energy data, at least in principle. However, non-negligible U_L rotations can lead to the vanishing of \hat{m}_{ee} and d_{12} at the level crossing energy scale, even though m_{ee} and $(m_{ee}m_{\mu\mu} - m_{e\mu}^2)$ are not vanishing at low energy.

Between the GUT and the seesaw scales one has to consider the evolution of the neutrino Yukawa couplings h and of the Majorana mass matrix of RH neutrinos M_R rather than the evolution of the effective matrix m [58, 59, 60, 61]. We assumed that, at the GUT scale, h is related with the Yukawa couplings of quarks or charged leptons. The evolution of h with decreasing mass scale will not modify the hierarchy $m_{D1} \ll m_{D2} \ll m_{D3}$, and its effects can be absorbed into a redefinition of our indicative values of $m_{D1,D2,D3}$.

The RGE effects on M_R are due to the neutrino Yukawa couplings; they can, in principle, be important in the cases of strongly degenerate RH neutrinos. Consider the stability of the structure of M_R in the special case that leads to a successful leptogenesis. Recall that in this case the (12)-sector of RH neutrinos is characterized by $M_{1,2} \sim 10^8$ GeV, $(M_2 - M_1)/M_1 \lesssim 10^{-5}$ and $\Delta \approx \pi/2$, where Δ is defined in Eq.(35). The largest correction to the (12)-block of M_R between M_{GUT} and $M_{1,2}$ is the correction to the 22-element:

$$\frac{(\Delta M_R)_{22}}{(M_R)_{22}} \sim \frac{m_{D2}^2}{16\pi^2 v^2} \log \left(\frac{M_{\text{GUT}}}{10^8 \text{ GeV}} \right) \approx 6 \cdot 10^{-7} \left(\frac{m_{D2}}{0.4 \text{ GeV}} \right)^2 .$$

Therefore, the radiative corrections cannot generate a relative splitting between M_1 and M_2 exceeding 10^{-5} . Moreover, at one loop level, the phases of $(M_R)_{ij}$ have no RGE evolution and so the relation $\Delta \approx \pi/2$ is not modified.

It has been recently shown [62] that, assuming exact degeneracy of M_1 and M_2 at the GUT scale, successful leptogenesis can be realized thanks to the radiatively induced splitting at the scale $M_1 \approx M_2$.

4) Large left-handed Dirac-type mixing.

Let us abandon the hypothesis $U_L \approx \mathbb{1}$. If the matrix U_L is arbitrary, the connection between the low energy data and the structure of M_R is weakened. This additional freedom relaxes the phenomenological constraints on RH neutrinos. In fact, now the unique low energy requirement on the seesaw mechanism is to reproduce the light neutrino masses, given by the eigenvalues of \hat{m} (Eq.(4)); the correct leptonic mixing matrix U_{PMNS} can always be obtained through the proper choice of U_L . As an example, let us consider

the case of non-degenerate RH masses and take the following RH mixing matrix:

$$U_R = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot K. \quad (37)$$

The eigenvalues of the matrix $\hat{m} = -m_D^{diag} M_R^{-1} m_D^{diag}$ are given, approximately, by $m_{D2}^2/(4M_2)$, $m_{D2}^2/(2M_1)$, m_{D3}^2/M_3 . Taking $M_1 \approx 10^{10}$ GeV $\cdot (m_{D2}/0.4 \text{ GeV})^2$, M_2 a few times larger and $M_3 \approx 2 \cdot 10^{14}$ GeV $\cdot (m_{D3}/100 \text{ GeV})^2$, one can reproduce the solar and atmospheric mass squared differences in Eq.(8). Since \hat{m} is approximately diagonal, the solar and atmospheric mixing angles are generated by U_L , which should have an almost bimaximal form.

Notice that in this scenario we have $|\hat{m}_{ee}| \approx (m_{D1}/m_{D2})|\hat{m}_{e\mu}|$. In a sense this situation is intermediate between the generic case and the special case $\hat{m}_{ee} \rightarrow 0$ (compare with Eq.(17)). However it cannot be realized unless large rotations are allowed in U_L . Notice that, in the SUSY case, these large rotations can be excluded by future stronger bounds on LFV decays.

Replacing Eq.(37) into Eqs.(29) and (25), it is easy to calculate the washout mass parameter and the asymmetry produced in the decays of N_1 :

$$\tilde{m}_1 = \frac{m_{D2}^2}{2M_1} \approx \sqrt{\Delta m_{sol}^2}, \quad \epsilon_1 \approx \frac{3m_{D2}^2}{32\pi v^2} \sin(\phi_2 - \phi_1) \frac{M_1}{M_2}.$$

Taking $\phi_2 - \phi_1 \sim \pi/2$ (note that the CP-parities of N_1 and N_2 are not constrained in this case), we get

$$\eta_B \approx 3 \cdot 10^{-11} \frac{M_1}{M_2} \left(\frac{m_{D2}}{0.4 \text{ GeV}} \right)^2.$$

Thus, for a moderate hierarchy between M_1 and M_2 , a value of m_{D2} around a few GeV can lead to a successful leptogenesis. This example shows that, relaxing the hypothesis $U_L \approx \mathbb{1}$, it is easier to realize baryogenesis via leptogenesis. In particular, the degeneracy of the masses of RH neutrinos M_i is no longer necessary, but the hierarchy of M_i should not be as large as it is in the generic case.

5) Non-thermal leptogenesis.

Let us comment on the possibility of non-thermal production of the heavy RH neutrinos [63, 64, 65, 66, 67], that in principle can lead to a successful leptogenesis for values of the parameters M_1 and \tilde{m}_1 for which thermal leptogenesis does not work.

In fact, it is interesting that also non-thermal leptogenesis is strongly constrained in our framework. Consider the generic case. Since M_1 is relatively light ($\lesssim 10^7$ GeV), ϵ_1 is very small. Moreover, as \tilde{m}_1 is relatively large ($\gtrsim \sqrt{\Delta m_{sol}^2}$), thermal effects washout (at least partially) the asymmetry generated in the decays of non-thermally produced RH neutrinos [68]. As a consequence, even in the non-thermal case, the asymmetry generated by N_1 turns out to be insufficient and, to enhance it, one has to resort again to the special case $\hat{m}_{ee} \rightarrow 0$.

It is known, however (see, e.g., [69, 70]), that also the asymmetries generated by N_2 and/or N_3 can survive if (1) they are produced non-thermally at reheating and (2) N_1 is

not in thermal equilibrium at the reheating temperature T_{RH} . In fact, the asymmetries $\epsilon_{2,3}$ can be large (they are of the order of $m_{D2,D3}^2/(16\pi v^2)$ in the generic case and even larger in the special case $d_{12} \rightarrow 0$: $\epsilon_{2,3} \sim m_{D2}/m_{D3}$). However, partial thermalization of $N_{2,3}$ and subsequent washout can occur after reheating. Moreover, to avoid later cancellation of $\epsilon_{2,3}$, N_1 should not enter into thermal equilibrium at any temperature $T \lesssim T_{RH}$.

In this case an accurate computation of the final asymmetry would require to solve the complete set Boltzmann equations describing the evolution of the number densities of all three RH neutrinos and of $B - L$.

8 Conclusions

We have analyzed the structure of the RH neutrino sector in the framework of type-I seesaw mechanism. We have found a convenient parameterization in which the mass matrix of RH neutrinos, M_R , is a function of the low energy neutrino data, the neutrino Dirac-type masses m_{Di} and the left-handed Dirac-type mixing matrix U_L . Our analysis is based on the assumptions of hierarchical m_{Di} (by analogy with quark masses) and small mixing in U_L (by analogy with CKM mixing).

The presence of two large mixing angles (θ_{12} and θ_{23}) and the weak mass hierarchy ($\sqrt{\Delta m_{sol}^2/\Delta m_{atm}^2} \approx 0.2$) in the light neutrino sector lead, in general, to a “quasi-democratic” structure of the mass matrix m in the flavor basis, with values of all its elements within one order of magnitude of each other. This implies that M_R has a strong (nearly quadratic in m_{Di}) hierarchy of eigenvalues and small mixing. The lightest RH neutrino has a mass $M_1 < 10^6$ GeV. As a consequence, the predicted η_B is of the order of $\sim (10^{-16} - 10^{-14})$ and the scenario of baryogenesis via leptogenesis does not work.

We have identified the special cases which correspond to the level crossing points, when either two or all three masses of RH neutrinos are nearly equal. We have found two level crossing conditions:

- (1) $\hat{m}_{ee} \rightarrow 0$ ($N_1 - N_2$ crossing);
- (2) $d_{12} \equiv (\hat{m}_{ee}\hat{m}_{\mu\mu} - \hat{m}_{e\mu}^2) \rightarrow 0$ ($N_2 - N_3$ crossing).

In the crossing points the mixing of the corresponding neutrino states is maximal and their CP-parities are nearly opposite. The level crossing conditions can be realized in agreement with low-energy data, because of the freedom in the choice of light neutrino absolute mass scale m_1 and Majorana phases ρ and σ , as well as in the choice of small rotations in U_L .

The thermal leptogenesis can be successful only in the special case with vanishing \hat{m}_{ee} . It is characterized by $M_1 \approx M_2 \sim 10^8$ GeV, $M_3 \sim 10^{14}$ GeV and $(M_2 - M_1)/M_2 \lesssim 10^{-5}$. N_1 and N_2 are strongly mixed and their mixing with N_3 is very small. The CP-violating phase Δ in Eq.(35) should be very close to $\pi/2$. Notice that this unique case with a successful leptogenesis is defined very precisely. It has a number of characteristic features which can give important hints for model building.

We have discussed in detail the stability of this result. It turns out that the successful scenario works also in the case of SUSY. Moreover it is stable under radiative corrections. The approximate pseudo-Dirac structure can be motivated by some flavor symmetry operating in the RH sector.

Can the unique successful special case that we found be ruled out? Since it requires a

suppression of \hat{m}_{ee} , it will be excluded in case of a positive signal of neutrinoless 2β -decay with m_{ee} close to the heaviest of the light neutrino masses (which could be measured in direct neutrino mass search experiments). In fact, if m_{ee} takes this “maximal” value, than \hat{m}_{ee} cannot vanish unless mixings in U_L are very large. If the condition $\hat{m}_{ee} \rightarrow 0$ is not realized, one will be left with the following alternatives:

- The quark-lepton symmetry is strongly violated: there is no strong hierarchy of the eigenvalues of m_D ($m_{D1}/m_{D2}, m_{D2}/m_{D3} \gtrsim 10^{-1}$) and/or the Dirac-type left-handed mixing is large (U_L contains the solar and/or atmospheric mixings).
- Type-I seesaw is not the sole source of neutrino mass; the simplest alternative could be type-II seesaw [71, 72, 73, 74, 75, 76] in which there is an additional contribution from an $SU(2)_L$ -triplet Higgs. Another possibility is that the seesaw is not the true mechanism of neutrino mass generation.
- A mechanism other than the decay of RH neutrinos contributes to leptogenesis (for leptogenesis in the presence of an $SU(2)_L$ -triplet see [77, 78, 79]) or the Baryon Asymmetry of the Universe is generated through a different mechanism, which has nothing to do with leptogenesis.

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